1 Units for density-independent and density-dependence population dynamics

Dynamic population models generally take the form

$$\frac{\mathrm{d}N(t)}{\mathrm{d}t} = f(N(t)). \tag{1}$$

This ordinary differential equation is autonomous because the state variable, population density (N), does not explicitly depend on time, t; it explicitly depends on density. The entire system changes as a function of the population's own density, $f(N(t))$. Expanding the equation:

$$\frac{\mathrm{d}N(t)}{\mathrm{d}t} = \lim_{\Delta t \to 0} \frac{N(t+\Delta t) - N(t)}{(t+\Delta t) - (t)} = f(N(t)) \tag{2}$$

we can see the unit of eqn. 1: the numerator is density (units [D]) and the denominator is per unit time (units [T^{-1}]).

Given constant conditions and resources population will grow constantly. We can write this as

$$\frac{\mathrm{d}N(t)}{\mathrm{d}t} = rN(t). \tag{3}$$

Here, r, is the intrinsic rate of growth and has units [T^{-1}]. To explicitly make the growth proportional to resource availability, we can assume a closed system with F_T total resources (units [R]). Within the system some resources will be unavailable, F (units [R]), and some that will be occupied, cN. N is again the density, and c is the resource use per unit density (units [R][D^{-1}]). We can express the resources as:

$$F_T = F + cN. \tag{4}$$

We can then make the assumption that r changes linearly proportionally to the amount of available resources. Specifically:

$$r = bF. \tag{5}$$

The parameter b is the rate of capture per units of resource, with units [T^{-1}][R^{-1}]. Rearranging and substituting eqn. 4 into eqn. 3, we arrive at:

$$\frac{\mathrm{d}N(t)}{\mathrm{d}t} = b(F_T - cN(t))N(t) = bF_TN(t) - bcN(t)^2. \tag{6}$$

If we define $r = bF_T$ and $\alpha = bc$, then we arrive at the r-α logistic equation,

$$\frac{\mathrm{d}N(t)}{\mathrm{d}t} = rN(t) - \alpha N(t)^2. \tag{7}$$

The units for α are [D^{-1}], which is commonly referred to as the 'crowding coefficient' because per-density (synonymous with per-capita) effect of density on the population growth rate.

If we wish to derive the r-K logistic equation, we want to set an *a priori* equilibrium point for the maximum density sustained by the ration of the total amount of resources to the resource use per-unit density; e.g., F_Tc^{-1}. Factoring out bF_Tc^{-1} from eqn. 6, we arrive at:

$$\frac{\mathrm{d}N(t)}{\mathrm{d}t} = bF_TN(t)\left(\frac{\frac{F_T}{c} - N(t)}{\frac{F_T}{c}}\right). \tag{8}$$

Substituting K (units [D][T^{-1}]) for F_Tc^{-1} and again $r = bF_T$ yields the r-K logistic equation:

$$\frac{\mathrm{d}N(t)}{\mathrm{d}t} = rN(t)\left(\frac{K - N(t)}{K}\right). \tag{9}$$

2 Table with parameters, units, and dimensions

Parameter	Units	Dimension
t	[T]	time
N	[D]	density
r	[T^{-1}]	intrinsic rate of growth
b	[T^{-1}][R^{-1}]	per-resource rate of capture
F_T	[R]	total system resources
F	[R]	available resources
c	[R][D^{-1}]	captured resource per individual
α	[D^{-1}]	per-capita effect of density
K	[D]	density